Discussion of

Limited Attention and the Dynamics of Probability Weighting

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@BFWG 2021

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Overview

- An ambitious (and nice) paper. Two parts
  1. Based on survey evidence, proposes **limited attention-probability weighting (LAPW) belief formation:** overweighting extreme realizations
     - The paper uses a convenient model to capture this, and show that it generates probability weighting and etc (akin to prospect theory)
  2. **Cross-sectional stock return predictability:** sorting on subjective LAPW-based Sharpe ratio predicts future returns

- I will discuss the two parts in order

1. The “limited attention probability weighting” belief formation mechanism

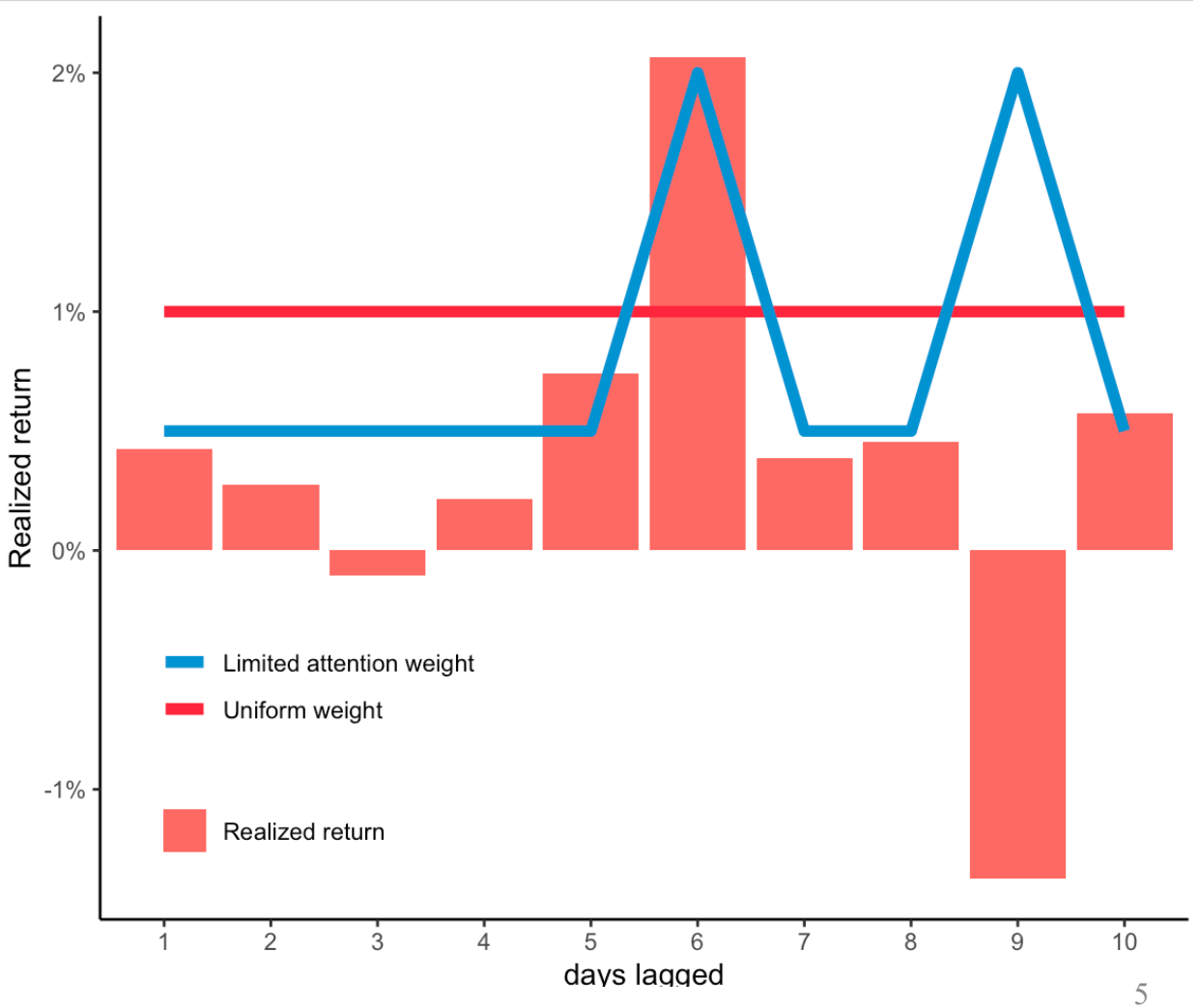
Background: extrapolative beliefs

- Greenwood Shleifer (RFS 2014), Cassella and Gulen (RFS 2018): investors have extrapolative expectations
  - They become optimistic (pessimistic) after experiencing good (bad) returns
  - Extrapolative expectations negatively predict aggregate stock returns

- How do investors weigh past return observations in belief formation?
  - Decaying weights: more recent observations are given more weights
    - GS RFS 2014: look-back half-life to be around one quarter

This paper: extreme realizations matter more

- **Order past *n* day returns, and the top/bottom ones are given more weight**
  - Very intuitive!
- Possible microfoundation from Kominers et al. (2018):
  - A rational agent with attention constraints may choose to pay more attention to the extreme realizations



Also, possible microfoundation from psychology

Money, Kisses, and Electric Shocks: On the Affective Psychology of Risk

Yuval Rottenstreich, Christopher K. Hsee

First Published May 1, 2001 | Research Article | Find in PubMed
https://doi.org/10.1111/1467-9280.00334

Article information

Altmetric 48

Abstract

Prospect theory's S-shaped weighting function is often said to reflect the psychophysics of chance. We propose an affective rather than psychophysical deconstruction of the weighting function resting on two assumptions. First, preferences depend on the affective reactions

I thank Lawrence Jin for this reference

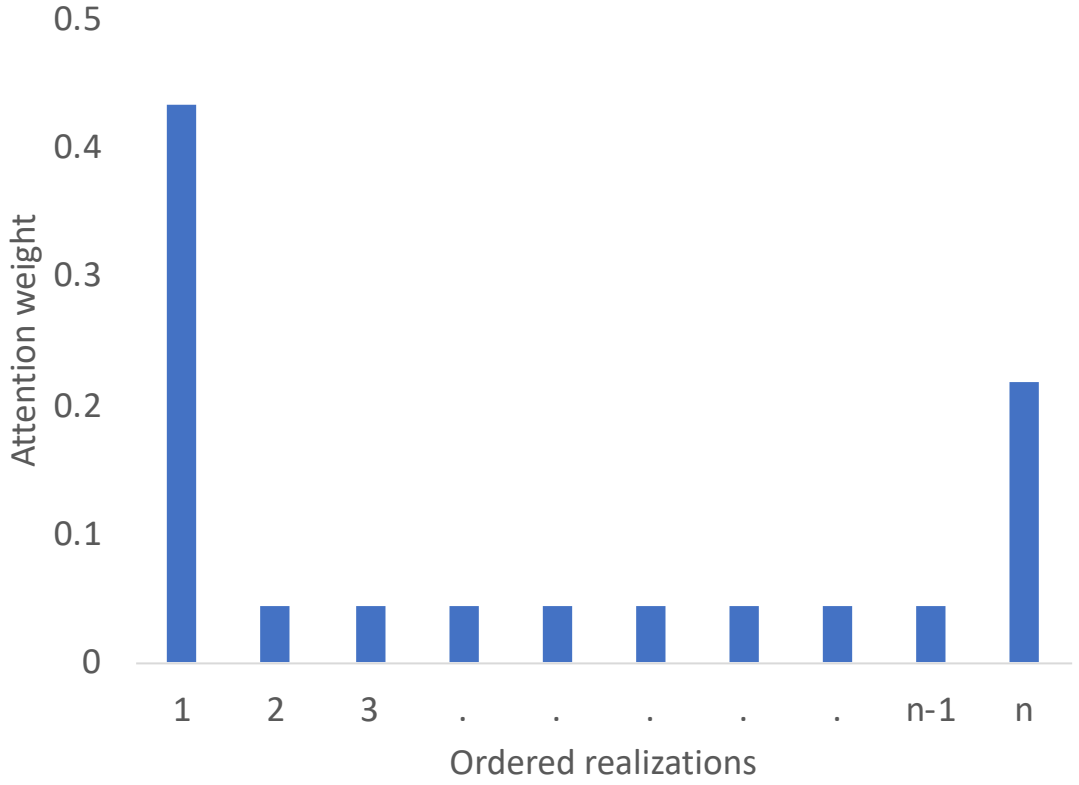
Nice empirical evidence based on surveys

- Investor beliefs depend more on the **smallest** and **largest** recently realized return

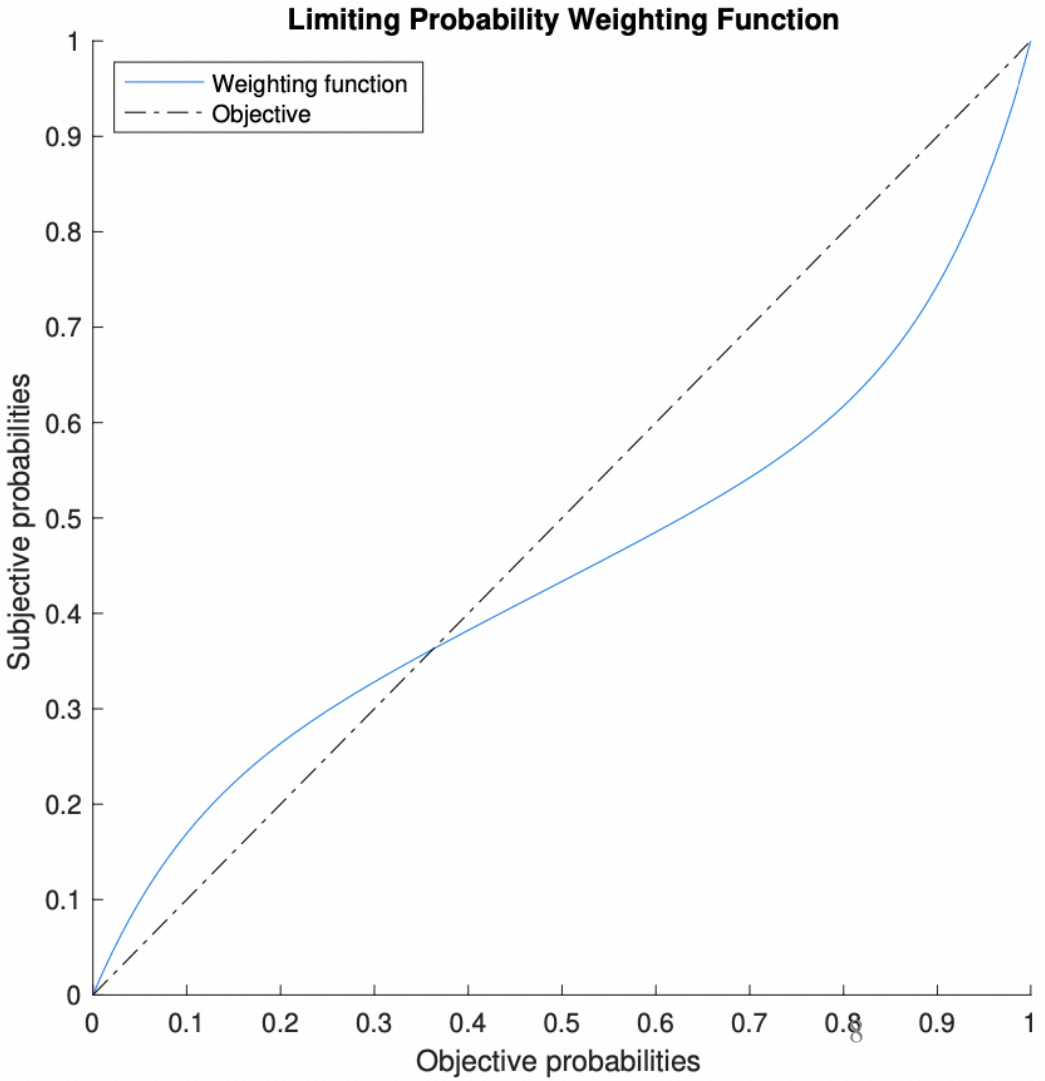
	Retail Investors (AAII-survey)				Institutional Investors (II-data)			
	(1) ΔBullish	(2) ΔBearish	(3) ΔBullish	(4) ΔBearish	(5) ΔBullish	(6) ΔBearish	(7) ΔBullish	(8) ΔBearish
$r_{t,1:6}$	2.92*** (3.98)	-2.48** (-2.10)	2.88*** (4.02)	-2.42** (-2.10)	1.97*** (5.38)	-1.37*** (-3.83)	2.00*** (5.59)	-1.40*** (-4.04)
$r_{t,2:6}$	3.31** (2.25)	-4.44** (-2.15)	3.44** (2.35)	-4.60** (-2.26)	0.96 (1.54)	-0.57 (-1.11)	0.79 (1.26)	-0.37 (-0.74)
$r_{t,3:6}$	1.95 (1.36)	-3.17* (-1.72)	1.41 (0.99)	-2.51 (-1.44)	1.52* (1.87)	-0.96 (-1.27)	1.07 (1.33)	-0.45 (-0.62)
$r_{t,4:6}$	1.53 (1.03)	-3.20 (-1.62)	0.47 (0.32)	-1.91 (-0.93)	0.15 (0.20)	-1.62** (-2.52)	-0.64 (-0.89)	-0.74 (-1.14)
$r_{t,5:6}$	1.87 (1.33)	-3.44* (-1.77)	1.26 (0.94)	-2.69 (-1.49)	1.41*** (2.74)	-0.65 (1.26)	1.12** (2.22)	-0.32 (-0.62)
$r_{t,6:6}$	2.30*** (2.81)	-2.24** (-1.99)	2.36*** (3.01)	-2.32** (-2.17)	1.85*** (6.42)	-1.63*** (-6.08)	1.85*** (6.73)	-1.63*** (-6.06)

Implication: limited attention => Prospect Theory-type probability weighting

- Suppose we use this weighting function…
  - Depends on 3 parameters



- Then we get PT-type probability weighting:

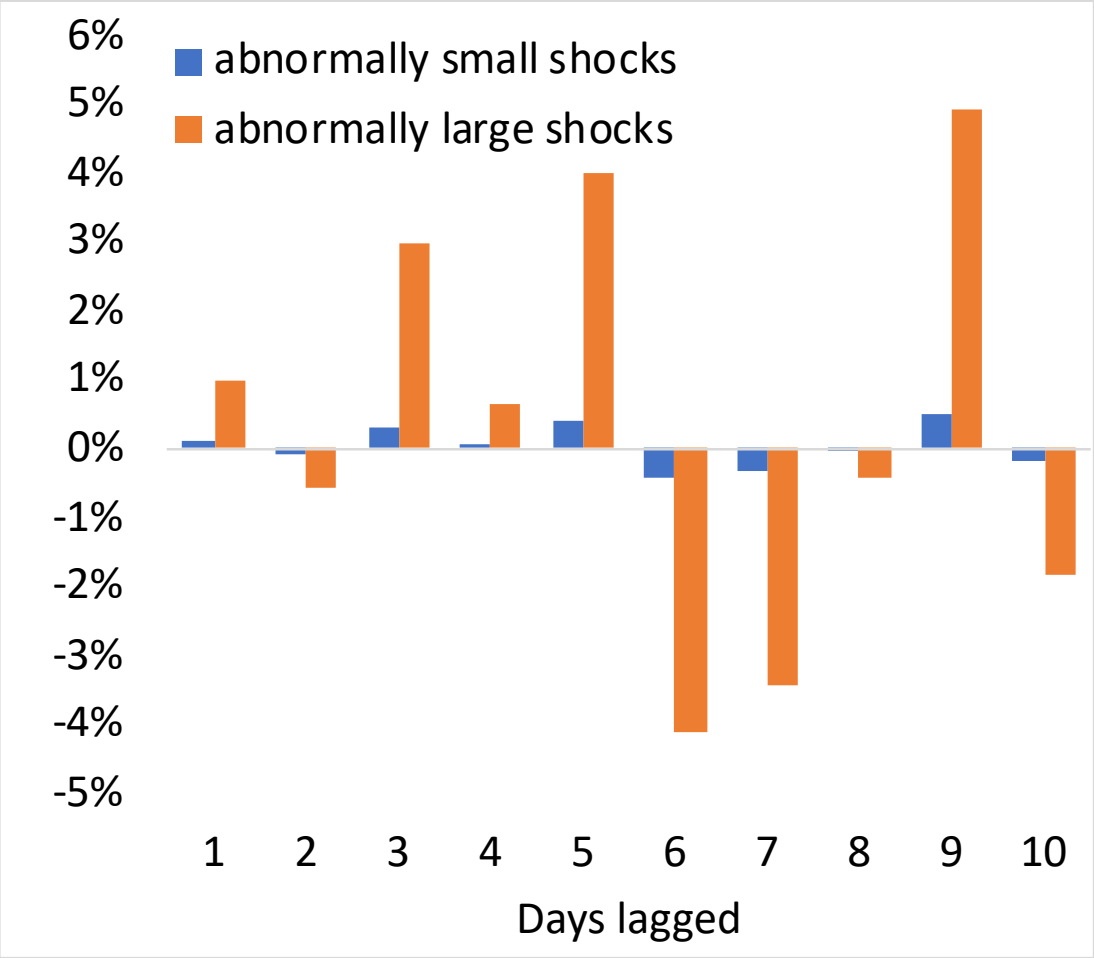


Discussion: difference from a close alternative?

- This paper: weights depend on relative ranking within recent observations
- Alternatively, investors may pay more attention to shocks that are large relative to some prior distribution?
  - Theory: Abel Eberly Panageas (EMA 2013), or more generally, the (S,s) models in sticky price models.
  - Empirical: Da Gurun Warachka (RFS 2014)
- I'll now illustrate their differences.

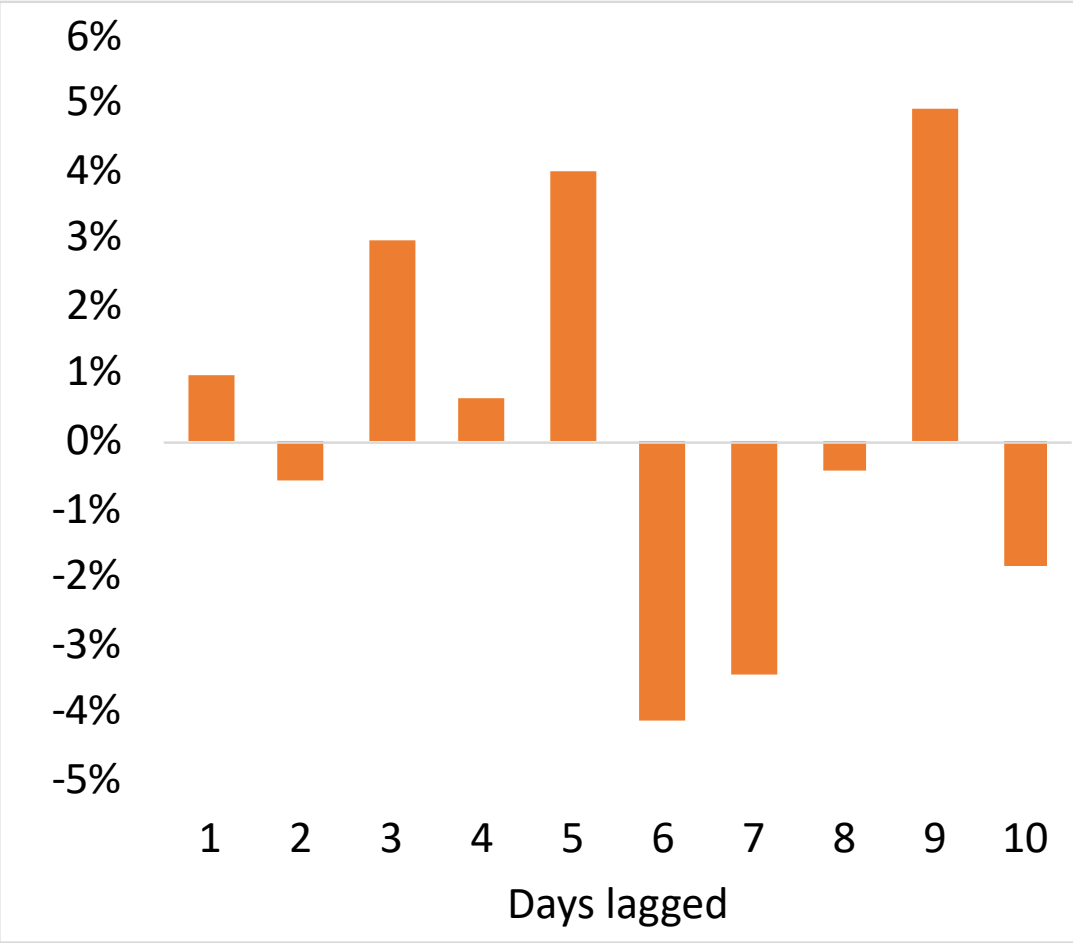
Difference 1: extreme observations, relative to **what**?

- Suppose stock returns are i.i.d. N(0,1%) distributions.



- The orange series = the blue series multiplied by 20
  - The blue series represent abnormally small shocks (normally, 1SD = 1%)
  - The orange series represent abnormally large shocks
- This paper: investors use the same attention weights in both cases
- The alternative model:
  - Investors goes to sleep when experiencing the blue series, but learns like crazy when experiencing orange series

Difference 2: just ordering, or also numerical values?



- Take this series of realizations as an example.
- This paper: investor will place more weight on 6th lag (min realization) and 9th lag (max realization)
- Alternative model:
  - The 7th lag will be assigned similar weight to the 6th, because the difference between them is small
  - In other words, *numerical values* matter

Bottom line: what determines attention weights?

- Ranking, as hypothesized by this paper, is reasonable
  - Caveat: existing ranking-based evidence are about cross-sectional comparisons, and not intertemporal comparisons (papers by Michael Ungeheuer; Kaniel-Parham (JFE 2017))
- The (S,s) view of comparing against priors is also reasonable
  - Suppose one experiences a sequence of 0% returns, and 0.001% in one day. Does she really overweight the latter? Does she not go to sleep?
  - Comparing realizations against some pre-existing priors is also reasonable

Empirically differentiating the two views

- For instance, take the evidence in Table 3:

$$Belief_{i,t} = \sum_{\tau=1,\ldots,6} \beta_\tau \cdot Ret_{i,t}^{ordered\ \tau^{th}}$$

- However, suppose the true model is (S,s):

$$Belief_{i,t} = \sum_{k} b\left(\left|Ret_{i,t-k}\right|\right) \cdot Ret_{i,t-k}$$

  - where $b(\cdot)$ is an increasing function

- I believe the authors will also find results qualitatively similar to theirs.
  - Important: even if alternative model is right, I still think the authors are making a contribution. I'm not aware of shock-size dependent evidence on belief extrapolation

2. Return predictability exercise

Test: predicting the cross-section of returns

- **Idea:** if investors trade according to their beliefs --- which are based on probability-weighted past returns --- their trades will create price pressures that subsequent revert

- The authors use one set of attention-weight parameters to compute the stock-level subjective Sharpe ratio over the past 30 days

- They then sort stocks using this measure and predict returns
  - This test is similar to that in Barberis Mukherjee Wang (RFS 2016), with a major difference that BMW looked at long past history (5 years)

Results (annualized returns)

		P1 Low LA	P2	P3	P4	P5	P6	P7	P8	P9	P10 High LA	P1-P10
Excess Return	EW	**0.1373** (5.13)	**0.1232** (4.52)	**0.1193** (4.53)	**0.1117** (4.22)	**0.0874** (3.45)	**0.0822** (3.18)	**0.0561** (2.20)	**0.0492** ()1.89	0.0411 (1.57)	0.0118 (0.48)	**0.1241** (8.30)
	VW	**0.1694** (5.47)	**0.1983** (4.68)	**0.1257** (3.22)	**0.1438** (3.56)	**0.1357** (3.72)	**0.0957** (2.60)	0.0409 ()1.19	**0.0991** (2.63)	0.0592 (1.62)	0.0329 (0.93)	**0.1284** (5.83)
3-Factor alpha	EW	**0.0244** 2.34()	0.0101 (0.99)	0.0076 (0.89)	0.0001 (0.02)	**-0.0205** (-2.73)	**-0.0224** (-2.86)	**-0.0473** (-5.83)	**-0.0526** (-5.97)	**-0.0593** (-6.41)	**-0.0779** (-7.88)	**0.1102** (7.52)
	VW	**0.0274** 1.98)	0.0190 (1.30)	-0.0137 (-0.93)	0.0197 (1.65)	-0.0163 (-1.24)	**-0.0397** (-3.24)	**-0.0486** (-4.09)	**-0.0521** (-3.86)	**-0.0704** (-4.90)	**-0.0767** (-5.29)	**0.1164** (5.10)
4-Factor alpha	EW	0.0051 (0.34)	-0.0150 (-1.11)	-0.0071 (-0.50)	-0.0153 (-1.20)	**-0.0289** (-2.21)	**-0.0356** (-2.59)	**-0.0549** (-4.05)	**-0.0474** (-3.08)	**-0.0531** (-3.53)	**-0.0606** (-3.64)	**0.0695** (3.20)
	VW	-0.0067 (-0.30)	0.0014 (0.07)	**-0.0412** (-1.82)	0.0119 (0.77)	-0.0322 (-1.46)	**-0.0683** (-4.14)	**-0.0664** (-3.76)	**-0.0535** (-2.91)	**-0.0654** (-3.07)	**-0.0567** (-2.47)	**0.0666** (1.83)
5-Factor alpha	EW	0.0047 (0.31)	-0.0123 (-0.92)	-0.0088 (-0.61)	-0.0109 (-0.82)	**-0.0255** (-1.96)	**-0.0279** (-2.10)	**-0.0516** (-3.77)	**-0.0384** (-2.59)	**-0.0513** (-3.30)	**-0.0568** (-3.42)	**0.0648** (3.02)
	VW	-0.0084 (-0.38)	0.0000 (0.00)	**-0.0492** (-2.26)	0.0093 (0.58)	**-0.0335** (-1.65)	**-0.0629** (-3.91)	**-0.0677** (-3.85)	**-0.0420** (-2.40)	**-0.0526** (-2.63)	**-0.0579** (-2.48)	**0.0644** (1.84)
6-Factor alpha	EW	0.0063 (0.36)	-0.0075 (-0.42)	-0.0093 (-0.51)	-0.0118 ()-0.69	-0.0234 (-1.43)	-0.0267 (-1.61)	**-0.0412** (-2.53)	-0.0282 (-1.61)	**-0.0398** (-2.06)	**-0.0432** (-2.13)	**0.0515** (2.36)
	VW	-0.0096 (-0.37)	-0.0066 (-0.26)	**-0.0596** (-2.23)	-0.0060 (-0.30)	-0.0322 (-1.24)	**-0.0769** (-4.46)	**-0.0529** (-2.52)	-0.0310 (-1.36)	**-0.0491** (-1.93)	**-0.0524** (-1.94)	0.0546 (1.46)
BMW	EW	**0.0357** (1.65)	0.0230 (0.99)	0.0232 (0.98)	0.0184 (0.77)	0.0006 (0.03)	-0.0010 (-0.04)	-0.0238 (-0.95)	-0.0265 (-1.00)	-0.0305 (-1.14)	**-0.0451** (-1.77)	**0.0843** (7.73)
	VW	**0.0788** (2.64)	**0.0926** (2.37)	0.0473 (1.37)	0.0616 (1.48)	0.0392 (1.19)	0.0083 (0.22)	-0.0071 (-0.20)	0.0225 (0.59)	0.0002 (0.01)	-0.0003 (-0.01)	**0.0921** (4.83)

Returns decay quickly at the daily frequency

		Number of days between sorting and realization of daily return									
		1	2	3	4	5	6	7	8	9	10
6-Factor alpha	EW	**0.6173** (26.58)	**0.3755** (25.36)	**0.3099** (21.58)	**0.2438** (18.79)	**0.1928** (16.32)	**0.1714** (15.02)	**0.1457** (12.94)	**0,1169** (10.91)	**0.0979** (9.18)	**0.0944** (9.04)
	VW	**0.6173** (6.58)	**0.4030** (5.52)	**0.4303** (5.54)	**0.3289** (4.53)	**0.2420** (3.59)	**0.3361** (4.80)	**0.3036** (4.32)	**0.1686** (2.77)	**0.1255** (2.00)	0.0958 (1.58)

Main empirical concern: is this different from *short-term return reversal*?

- Examine the sorting variable:

$$SharpeRatio = \frac{\overline{Ret}^{subjective} - r_f}{\sigma^{subjective}}$$

  - $\overline{Ret}^{subjective}$ must be correlated with $\overline{Ret}^{objective}$, which is known to strongly predict returns (short-term reversal)
- Table 5 controls for the liquidity factor in Nagel (RFS 2012)
  - However, short-term reversal isn't a factor, but a characteristic. That is, it could be entirely idiosyncratic
  - I see no alternative besides running Fama-MacBeth regressions and directly controlling reversal measures
- Put another way, one must make sure the return predictability arises from the difference between $\overline{Ret}^{subjective}$ and $\overline{Ret}^{objective}$

Second order empirical concerns

- Why pick this specific set of parameters?
  - The authors have good belief survey data. They could easily estimate the parameters from survey data

- Relationship with DOX
  - The authors show that market-level LA-PW mean, volatility, and skewness help explain the variation of Cassella Gulen (RFS 2018) degree-of-extrapolation (DOX) measure
  - However, what is the mechanism?
    - I may simply have missed it

Summary

- I think the survey-based results are very interesting and may be enough to stand on itself
  - As the paper discussed, researchers have primarily examined retail surveys. Surveys about institutional beliefs are rare
  - The paper can do more to differentiate itself from alternative mechanisms

- I have concerns about the return predictability exercise.
  - If the survey results end up strong, perhaps this part is not necessary

- This is a very nice paper and I appreciate the opportunity to discuss